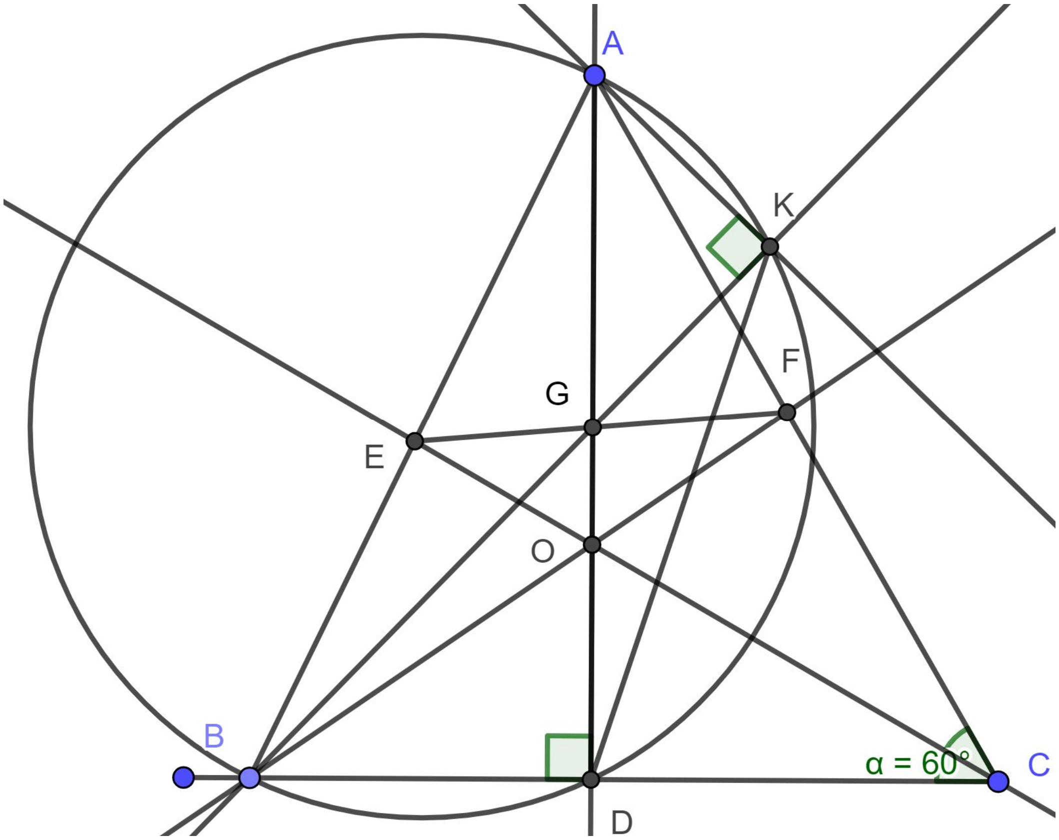
SOLUTION:



ABDK is a cyclic quadrilateral. (As $\angle AKB = 90^0,\ \angle ADB = 90^0$ on AB.)

Let us construct a circle passes through A. B, D and K.

Let L be the centre of the circle.

As AB is the diameter so L must lies on AB and AL = BL.

CE is angle bisector of $\angle ACB = 60^0 \Rightarrow \angle BCE = \angle ACE = 30^0$

Now $\angle CAD = \angle ACE = 30^0 \Rightarrow \angle CAO = \angle ACO$

$\Rightarrow AO = CO$ (Side opposite to equal angles of a triangle)

In triangle ODC, $\angle OCD = 30^0$

So, $OD = \frac{OC}{2} = \frac{AO}{2} \Rightarrow \frac{AO}{OD} = 2$ and $\frac{AO}{AD} = \frac{2}{3},\quad \frac{DO}{AD} = \frac{1}{3}$

As per concurrency theorem $\frac{AG}{OF} = \frac{AD}{OD} = 3$

$\Rightarrow \frac{AG}{OA} = \frac{3}{4} \Rightarrow AG = \frac{3}{4} \times OA = \frac{3}{4} \times \frac{2}{3} \times AD = \frac{AD}{2}$

so G is mid point of AD

$\Rightarrow$ **AG = GD --------(1)**

ΔGAB and ΔGKD are similar to each other, ($\angle BAG = \angle DKG,\ \angle ABG = \angle KDG$**)**

$\frac{AG}{KG} = \frac{BG}{DG} = \frac{AB}{KD}$ **----------------(2)**

ΔAGK and ΔBGD are similar to each other, ($\angle AGK = \angle BGD,\ \angle AKG = \angle BDG$**)**

$\frac{AG}{BG} = \frac{GK}{DG} = \frac{AK}{BD}$ **----------------(3)**

From (2) and (3) $\frac{AB}{KD} \times \frac{AK}{BD} = \frac{AG}{KG} \times \frac{GK}{DG}$

$\Rightarrow \frac{AB}{KD} \times \frac{AK}{BD} = \frac{AG}{DG}$ **-------------(4)**

$\Rightarrow \frac{AB}{KD} \times \frac{AK}{BD} = 1$ **By Using (1)**

$\Rightarrow AB \times AK = KD \times BD$ **----------(5)**

By Using (Ptolemy's theorem: If there is a quadrilateral inscribed in a circle, then the product of the diagonals is equal to the sum of the product of its two pairs of opposite sides.)

$AD \times BK = AK \times BD + AB \times KD$

$\Rightarrow BK^2 \times AD^2 = AK^2 \times BD^2 + AB^2 \times KD^2 + 2AK \times BD \times AB \times KD$ **(As** $AB \times AK = KD \times BD$**)**

$\Rightarrow BK^2 \times AD^2 = (AB^2 - BK^2) \times BD^2 + AB^2 \times KD^2 + 2AB^2 \times AK^2$

$\Rightarrow BK^2 \times AD^2 = AB^2 \times BD^2 - BK^2 \times BD^2 + AB^2 \times KD^2 + 2AB^2 \times AK^2$

$\Rightarrow BK^2 \times AD^2 + BK^2 \times BD^2 = AB^2 \times BD^2 + AB^2 \times KD^2 + 2AB^2 \times AK^2$

$\Rightarrow BK^2 \times (AD^2 + BD^2) = AB^2 \times BD^2 + AB^2 \times KD^2 + 2AB^2 \times AK^2$

$\Rightarrow BK^2 \times AB^2 = AB^2 \times BD^2 + AB^2 \times KD^2 + 2AB^2 \times AK^2$

$\Rightarrow BK^2 = BD^2 + KD^2 + 2AK^2$ **(Proved)**

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